

Claims 1 and 6



The path of a point fixed relative to a circle that rolls along a straight line is called a *trochood*. The easiest way to visualize this phenomenon is to think of the path of a reflector on a bicycle as someone is riding on a level street. The reflector rotates around the bub of the wheel, but yet the hub of the wheel is moving relative to the ground. Here is an apply that demonstrates this (without the bicycle) (NOTE: As of 10-500, I have rewritten the cycloid applet. I had received some comments suggesting that the old version caused some computers to crash. Hopefully the problem has been fixed. Let me know if you experience any further problems

Now think about what happens if the circle is instead rotating around another circle. In other words, the cyclist is now pedaling his ber way around the equator, instead of down the street. Mathematicians call this path an epicycloid. The rest of the world calls them SpiroGrapho!! The parametric equations for these curves are given by:

x(t)+(R+r)cos(t) - p\*cos((R+r)t/r)
y(t)+(R+r)cos(t) - p\*cos((R+r)t/r)

where R.t. and p me defined below. The applet below allows you to create all the SpinoGraphs your heart desires by varying the values of R. e and p, as well as the following parameters:

- · Radius l (R)
- Radius of circle (equator) centered at the origin.
- Radius 2 (r)

  Radius of circle (bicycle wheel) centered at (R-1,0)
- Position (p)
   Distance of Point (reflector) from the center of Circle2, the circle of radius r.
- Distance of Point (reflector) from the center of Circle 2, the circle of stadius s.

  Velocity
  Speed at which the SpiroGraph is drawn, with 0 being the slowest and 10 being the fastest. Can be adjusted while drawing is taking place. Sometimes half the fun is seeing your SpiroGraph being drawn! Sometimes not!

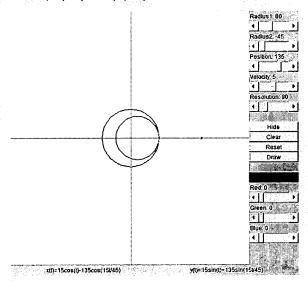
  Resolution
  Controls how precise the SpiroGraph is drawn. For instance, with a Resolution of 360, the points (a(t), y(t)) are plotted for t=0, 1, 2, 3, 6, 5, ..., 359, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 8, 10, ..., 359, 360 are used. Be careful, the larger the Resolution, the longer it will take to draw the SpiroGraph. Normally, a value between 270 and 360 will be good enough. With a small value of Radius 2 (say Radius 2=1), a small change in Resolution can result in remarkably different SpiroGraph.

  Hilder/Shew
  Changing the value of this button will determine whether or not to display the circles/saces while drawing is taking place. Note: SpiroGraphs are drawn much faster if you "Fide" the circles/saces.

- Clear
   This button will clear the screen of all SpiroGraphs.
- Reset
   This button stops the current SpiroGraph and returns Circle 2 and the Point to its original position
- These buttons should be self explanatory.

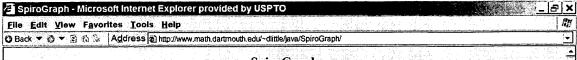
   Red/Green/Blue

You can adjust the values of these sliders to draw your SpiroGraph in virtually any color you like!



SIMPLE	Radiust	Radius2	Position	Resolution	Try this:
Cardioid	60	60	60	270	See what happens when you vary the postion to the left or to the right. How could you get a cardioid oriented differently?
Astroid	. 60	-15	-15	270	Another name for an astroid is a hypocycloid of four cusps. What would you change to get the same picture cotated 45 degrees? Vary the radius and try to get 3,5 or 6 cusps. Also, vary the position and see what happens.
Four-leaved	60	-15	45	270	Try to vary the radius and postion to get 3,5 or 6 leaves
Vertical Line	60	.30	.30	279	What would you change to get a horizontal line?
Elipse	50	-30	-90	270	What would you have to change in order to get an ellipse oriented horizontally? How would you get an ellipse on the inside of the fixed circle?
Rounded Square	60	-45	-191	270	How about a counded triangle or pentagon?
Gold fish	75	-25	85	270	This is supposed to be fun, right?

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The path of a point fixed relative to a circle that rolls along a straight line is called a prochold. The easiest way to visualize this phenomenon is to think of the path of a reflector on a bicycle as someone is riding on a level street. The reflector rotates around the hab of the wheel but yet the hab of the wheel is moving relative to the ground. Here is an applic that demonstrates this (without the bicycle) (NOTE: As of 10/6/00, I have rewritten the cycloid applet. I had received some comments suggesting that the old version caused some computers to crash. Hopefully the problem has been fixed. Let me know if you experience any further problems.

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where R. r, and p are defined below. The applet below allows you to create all the SpiroGraphis your heart desires by varying the values of R. c and p, as well as the following parameters:

- Radius 2 (R)
   Radius of circle (equator) centered at the origin.
   Radius 2 (r)
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- RABIUS OF LINES (ON.) 2500

  Position (p)

  Distance of Point (reflector) from the center of Circle 2, the circle of radius s.
- ocity
  Speed at which the SpiroGraph is drawn, with 0 being the slowest and 10 being the fastest. Can be adjusted while drawing is taking place. Sometimes half the fun is seeing your SpiroGraph being drawn! Sometimes not!

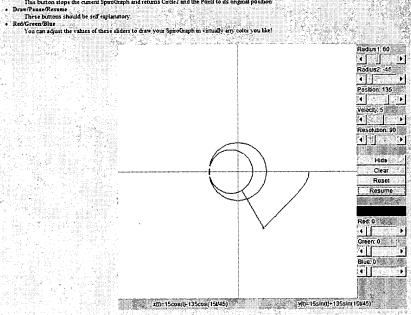
drawn! Sometimes not!

Resolution

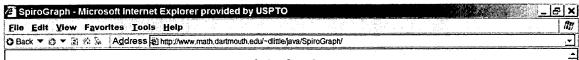
Controls how precise the SpiroGraph is drawn. For instance, with a Resolution of 560, the points (a(t), y(t)) me plotted for t=0, 1, 2, 3, 4, 5, ..., 359, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 359, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 359, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 359, 360 degrees. With a Resolution of 180, only the value of the small value of Redins 2 (say Redins 2-1), a small change in Resolution can result in remarkably different SpiroGraph. Normally, a value between 270 and 360 will be good enough. With a small value of Redins 2 (say Redins 2-1), a small change in Resolution can result in remarkably different SpiroGraph. Normally, a value between 270 and 360 will be good enough. With a small value of Redins 2 (say Redins 2-1), a small change in Resolution can result in remarkably different SpiroGraph. Normally, a value between 270 and 360 will be good enough. With a small value of Redins 2 (say Redins 2-1), a small change in Resolution of 180, only the longer in Will also to draw the SpiroGraph. Normally, a value between 270 and 360 will be good enough. With a small value of Redins 2 (say Redins 2-1), a small change in Resolution of 180, only the longer in Will also to draw the SpiroGraph. Normally, a value between 270 and 360 will be good enough. With a Small value of Redins 2 (say Redins 2-1), a small change in Resolution of 180, only the longer in Will also to draw the SpiroGraph. Normally, a value between 270 and 360 will be good enough. With a Small value of Redins 2 (say Redins 2-1), a small change in Resolution of 180, only the longer in Resolution of 180, only the small value of the Small value of Redins 2 (say Redins 2-1), a small change in Resolution of 180, only the small value of 180, only the smal

- Clear
   This button will clear the screen of all SpiroGraphs.

- Reset
  This button stops the current SpiroGraph and returns Circle? and the Point to its original position



CURVES	Since in		ha amount	***************************************	
Cardioid	60	60	60	270	See what happens when you vary the postion to the left or to the right. How could you get a cardioid oriented differently?
Astroid	60	-15	-15	270	Another name for an astroid is a hypocycloid of four cusps What would you change to get the same picture rotated 45 degrees? Yary the radius and try to get 3,5 or 6 cusps. Also vary the position and see what happens.
Four-leaved rose	60	-15	45	270	Try to vary the radius and postion to get 3,5 or 6 leaves
Vertical Line	60	-30	-33	270	What would you change to get a horizontal line?
Ellipse	60	-30	-90	270	What would you have to change in order to get an ellipse oriented horizontally? How would you get an ellipse on the inside of the fixed circle?
Rounded Square	60	-45	-101	279	How about a counded triangle or pentagon?
Gold fish	75	-25	85	279	This is supposed to be fun, right?



The path of a point fixed relative to a circle that rolls along a straight line is called a prochold. The easiest way to visualize this phenomenon is to think of the path of a reflector on a bicycle as someone is siding on a level street. The reflector rotates around the hub of the wheel, but yet the hub of the wheel is moving relative to the ground. Here is an apply that demonstrates this (without the bicycle) (NOTE: As of 10/5/09, I have rewritten the cycloid applet. I had received some comments suggesting that the old version caused some computers to crash. Hopefully the problem has been fixed. Let me know if you experience any further problems.)

Now think about what happens if the circle is instead rotating around another circle. In other words, the cyclist is now pedating his her way around the equator, instead of down the street. Mathematicians call this path an epicycloid. The rest of the world calls them SpiroGrapha!! The parametric equations for these curves are given by:

x(t)=(R+r)cos(t) - p\*cos({R+r}t/r) y(t)=(R+r)cin(t) - p\*sin({R+r}t/r)

where R.r., and p are defined below. The applet below allows you to create all the SpiroGraphs your heart desires by varying the values of R. e and p, as well as the following parameters:

- Radius I (R)
   Radius of circle (equator) centered at the origin.

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  Radius: 07

  Radius of circle (bicycle wheel) centered at (R=1,0)

  Position (p)

  Distance of Point (reflector) from the center of Circle2, the circle of radius r.

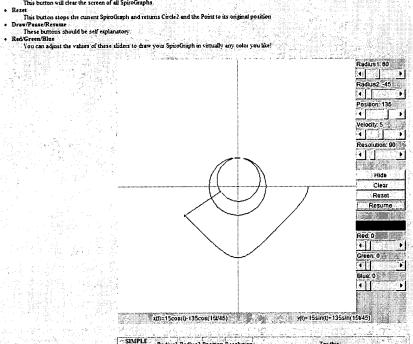
Distance of Point (reflector) from the center of Circle 2, the circle of radius r.

Velocity
Speed at which the SpiroGraph is drawn, with 0 being the slowest and 10 being the fastest. Can be adjusted while drawing is taking place. Sometimes half the fun is seeing your SpiroGraph being drawn! Sometimes not!

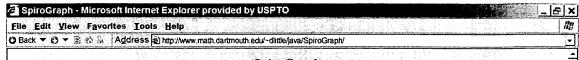
Resolution
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Clear

- Clear
   This button will clear the screen of all SpiroGraphs.



~ SIMPLE CURVES	Radiusl	Radius2	Position	Resolution	Try this:
Cardioid	60	60	60	270	See what happens when you vary the postion to the left or to the right. How could you get a cardioid oriented differently?
Astroid	60	-15	-15	270	Another name for an astroid is a hypocycloid of four cusps. What would you change to get the same picture rotated 45 degrees? Vary the radius and try to get 3,5 or 6 cusps. Also, vary the position and see whar happens.
Four-leaved	60	-15	45	270	Try to vary the radius and postion to get 3,5 or 6 leaves
Vertical Line	60	-30	.30	270	What would you change to get a horizontal line?
Ellipse	50	-30	-90	270	What would you have to change in order to get an ellipse oriented horizontally? How would you get an ellipse on the inside of the fixed circle?
Rounded Square	60	-45	-101	270	How about a rounded triangle or pentagon?
Gold fish	] 75	-25	85	270	This is supposed to be fun, right?



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- Radius I (R)
   Radius of circle (equator) centered at the origin.

- RAGMIS OF CEPCE (ESQUART) CTRICATE ON THE MEMORY OF RAGINES (P. C)

  RAGINES (P. C)

  RAGINES OF CEPCE Wheel) Centered at (R-1,0)

  Distance of Point (reflector) from the center of Circle2, the circle of radius r.

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  \* Velocity

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  \* Hilde: Show

  Chapping the value of this button will determine whether or not to display the circles (saye while drawing is taking place. Note: SpiroGraph, or a than much faster if you "Hide" the circles (saye while drawing is taking place.
- 10-2008"
  Changing the value of this buston will detenuine whether or not to display the circles uses while drawing is taking place. Note: SpiroGraphs are drawn much faster if you "Hide" the circles uses. Changing the vacce.

  • Clear

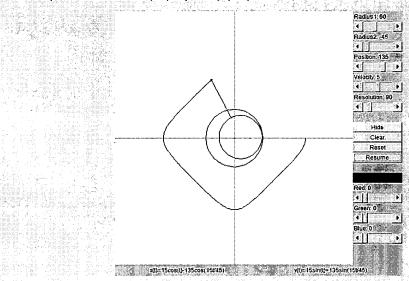
  This button will clear the screen of all SpiroGraphs.

- Reset
   This button stops the current SpiroGraph and returns Circle? and the Point to its original position

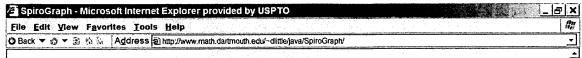
- This button tops use curren symbols;

  DrawPauseResume
  These buttons should be self explanatory.

  RedGreen/Blue
  You can adjust the values of these sliders to draw your SpiroGraph in virtually any color you like!



SIMPLE CURVES	Rediusl	Radius2	Position	Resolution	n Try this
Cardioid	60	60	60	270	See what happens when you vary the postion to the left or to the right. How could you get a cardioid oriented differently?
Astroid	60	-15	-15	279	Another name for an astroid is a hypocycloid of four cusps. What would you change to get the same picture rotated 43 degrees? Vary the radius and try to get 3,5 or 6 cusps. Also, vary the position and see what happens.
Four-leaved rose	60	-15	45	278	Try to vary the radius and postion to get 3,5 or 6 leaves
Vertical Line	60	-30	.30	270	What would you change to get a horizontal line?
Ellipse	50	-30	-90	270	What would you have to change in order to get an ellipse oriented horizontally? How would you get an ellipse on the inside of the fixed circle?
Rounded Square	60	-45	-101	270	How about a rounded triangle or pentagon?
Gold fish	75	-25	\$5	270	This is supposed to be fun, right?



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where R., and p me defined below. The applet below allows you to create all the SpiroGraphu your heart desires by varying the values of R, 1 and p, as well as the following parameters:

- Radius 1 (R)
  Radius of circle (equator) centered at the origin.
  Radius 2 (r)
  Radius of circle (bicycle wheel) centered at {R=r,0}.
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   Distance of Point (reflector) from the center of Circle2, the circle of radius r.

Distance of rout retrictory from two cents of control was control.

Velocity

Speed at which the SpiroGraph is drawn, with 0 being the slowest and 10 being the fastest. Can be adjusted while drawing is taking place. Sometimes half the fun is seeing your SpiroGraph being drawn! Sometimes not!

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Resilentian
Controls how precise the SpiroGraph is drawn. For instance, with a Resolution of 360, the points (x(t), y(t)) are plotted for t=0, 1, 2, 3, 4, 5, ..., 359, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 355, 360 are used. Be cueful, the larger the Resolution, the longer it will take to draw the SpiroGraph. Normally, a value between 270 and 360 will be good enough. With a small value of Radius 2 (say Radius 2=1), a small change in Resolution can result in remarkably different SpiroGraphs.

Hide/Show

- re-nown

  Changing the value of this button will determine whether or not to display the circles/sees while drawing is taking place. Note: SpiroGraphs are drawn much faster if you "hide" the circles/sees. Clear
   This button will clear the screen of all SpiroGraphs.

- This button suggested as a screen of the second of the Point to its original position.

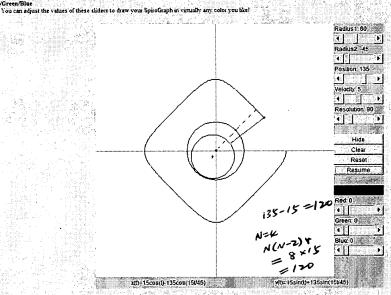
  This button stops the current SpiroGraph and returns Circle? and the Point to its original position.

  DrawPause/Returne

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  Red/Green/Blue

  Value and state the values of these dides to draw your SpiroGraph in variably any color you bite!



SIMPLE	Radius l	Radius2	Position	Resolution	Try this:
Cardioid	60	60	60	270	See what happens when you vary the postion to the left or to the right. How could you get a cardioid oriented differently?
Astroid	60	-15	-25	270	Another name for an astroid is a hypocycloid of four cusps. What would you change to get the same picture rotated 45 degrees? Vary the radius and try to get 3,5 or 6 cusps. Also vary the position and see what happens.
Four-leaved rose	60	-15	45	270	Try to vary the radius and postion to get 3,5 or 6 leaves
Vestical Line	60	-30	.30	270	What would you change to get a horizontal line?
Ellipse	8	-30	-90	270	What would you have to change in order to get an ellipse oriented horizontally? How would you get an ellipse on the inside of the fixed circle?
Rounded Square	60	-45	.191	270	How about a counded triangle or pentagon?
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- Radius of circle (equator) centered at the origin.

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- NAMES of CRUE (MESSAGE STATES)

  Postfilm (9)

  Distance of Point (reflector) from the center of Circle 2, the circle of radius s.

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Velocity
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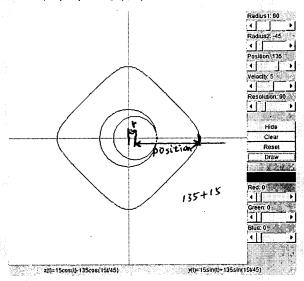
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Hide-Show
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Clear

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- Red/Green/Blue

You can adjust the values of these sliders to thaw your SpiroGraph in virtually any color you like!



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Astroid	60	-15	-15	270	Another name for an astroid is a hypocycloid of four cusps. What would you change to get the same picture rotated 45 degrees? Vary the radius and try to get 3,5 or 6 cusps. Also vary the position and see what happens.
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Ventical Line	60	.30	-30	270	What would you change to get a horizontal line?
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